

Assuming that the acoustic disturbances are, in this case, small in comparison with the vortical (3) and entropic (2) ones, from the diagram (Fig. 6) one can determine the intensity of the vortical mode  $\langle u \rangle = 0.64\%$  and entropic mode  $\langle T \rangle = 0.20\%$  with a correlation coefficient between them  $R_{uT} = -0.73$ . The measurements with the constant-temperature anemometer (for one overheat ratio) do not allow one to obtain any information about fluctuations except approximate values of the intensity of the mass flow rate fluctuations  $\langle m \rangle \approx \vartheta(r)/r = 0.87\%$ . Consequently, for high subsonic velocities, it is advisable to use constant-current anemometers in order to obtain fluctuation diagrams and to separate modes of disturbances. Then, the form of the diagrams for the vortical and entropic modes is analogous to the diagram for supersonic velocities, and the diagram of acoustic disturbances may consist of two elements of a straight line intersecting at the abscissa.

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#### ASYMPTOTIC ANALYSIS OF FLOW INSTABILITY IN A COMPRESSIBLE BOUNDARY LAYER ON A CURVED SURFACE

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A nonviscous instability mechanism exists in a two-dimensional boundary layer on a curved surface, as a result of which there appear pairs of vortices oriented along the flow and rotating in opposite directions. These are commonly termed Görtler vortices (Fig. 1). With increase in intensity of these vortices down the flow they may cause a transition of the laminar boundary layer into a turbulent one. In experiments in the boundary layer transition region they manifest themselves as periodically distributed thermal fluxes, shear stresses, etc., in a direction transverse with respect to the direction of the main flow in the boundary layer (see, for example, [1, 2], and the bibliography presented in [3]). There is interest in boundary layer stability on a curved surface because the supercritical profiles which have been designed in the past have segments with quite large curvature, so that a transition to a turbulent boundary layer develops under the action of centrifugal forces [4]. Aside from this, the effect of surface curvature on the character of the flow in the boundary layer requires special attention in design of nozzles for low noise supersonic tubes [5]. Reviews of preceding studies of boundary layer instability on curved surfaces were presented in [3, 6, 7]. In our opinion, the main unique feature of this problem is that in comparison to Tollmin-Schlichting wave instability or instability of secondary flows in a boundary layer in the vicinity of an edge in arrowlike wings, Görtler vortices are characterized by a weak intensity of motion and relatively slow intensification down the flow. Hence in analyzing the flow stability in the general case it is necessary to preserve those terms

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of the linearized Navier-Stokes equations which are usually related to the effects of the nonparallel nature of the flow. On this question we must note the studies [8-10], which considered the effect of boundary layer suction and cooling, as well as that of the pressure gradient on flow stability. In [11] an asymptotic expression was obtained for the neutral stability curve for the case of an incompressible boundary layer, where the spatial period of the vortices in the transverse direction is small in comparison to the boundary layer thickness. The problem of boundary layer stability on a curved surface was analyzed in [12] within the framework of a numerical solution of equations in partial derivatives. Attention was called to the fact that the two-term asymptotic expression found for the neutral stability curve proved convenient over a quite wide range of the parameter characterizing the linear size of the vortices in the transverse direction. The present study will present a generalization of the results of [11] to the case of Görtler vortices increasing in intensity down the flow in a compressible boundary layer, which permits derivation of simple approximate relations which can be used in engineering practice.

1. Formulation of the Problem. We consider as the basic flow a two-dimensional compressible boundary layer on a slightly curved surface. Following [10], we will use a coordinate system  $(x, y, z)$  based on flow and equipotential lines for a nonviscous flow on a curved surface. The coordinate  $x$  is directed along the flow line, the  $y$  coordinate along the equipotential line, and the  $z$  coordinate, normal to the plane  $(x, y)$ . For the characteristic length scale we choose  $\delta = \sqrt{\nu_\infty x / U_\infty}$ , where  $U_\infty$  is the velocity of the incident flow;  $\nu_\infty$  is the kinematic viscosity coefficient in the incident flow. For the pressure scale we choose  $\rho_\infty U_\infty^2$  (where  $\rho_\infty$  is the density in the incident flow). Temperature will be measured in units of the incident flow temperature  $T_\infty$ . The problem to be analyzed is characterized by two small parameters:  $\epsilon_0 = R^{-1} = \sqrt{\nu_\infty / U_\infty x}$ , the viscous parameter;  $k = (K\delta)^{1/2}$ , the curvature parameter (where  $K$  is the curvature of the surface flowed over). We will consider the limit  $\epsilon_0, k \rightarrow 0$ , so that  $G = k/\epsilon_0 = \text{const}$  ( $G$  is the Görtler constant). Then as the basic approximation for the main flow we obtain the equations of a boundary layer on a planar plate. Concerning perturbations of the  $x$ -,  $y$ -, and  $z$ -components of the velocity  $u, v, w$ , the pressure  $p$ , and the temperature  $\Theta$ , we assume that they are of the following order of magnitude [10]:  $u = O(1)$ ,  $v = O(R^{-1})$ ,  $w = O(R^{-1})$ ,  $p = O(R^{-2})$ ,  $\Theta = O(1)$ .

These relationships between the defining parameters have been confirmed by experimental observations and direct numerical calculations [3]. If we choose as the length scale along the coordinate  $x$   $L = R\delta$  and seek a solution for the perturbations, having separated them by order of magnitude, in the form

$$\begin{aligned} u &= \widehat{u}(y) \cos \beta z \cdot \exp\left(\int \sigma dx\right), & v &= \widehat{v}(y) \cos \beta z \cdot \exp\left(\int \sigma dx\right), \\ w &= \widehat{w}(y) \sin \beta z \cdot \exp\left(\int \sigma dx\right), & p &= \widehat{p}(y) \cos \beta z \cdot \exp\left(\int \sigma dx\right), \\ \Theta &= \widehat{\Theta}(y) \cos \beta z \cdot \exp\left(\int \sigma dx\right), \end{aligned}$$

then from the linearized Navier-Stokes equations we obtain a system of ordinary differential equations [10] (there are errors in the expressions of [10] which can easily be seen from simple calculations). For the equations thus obtained we formulate boundary conditions:  $u, v, w, \Theta \rightarrow 0$  as  $y \rightarrow \infty$ ;  $u, v, w = 0$  at  $y = 0$ . If the surface flowed over is thermally isolated, then at  $y = 0$  the condition  $\partial\Theta/\partial y = 0$  must be satisfied. If the temperature of the surface flowed over has a fixed temperature, then at  $y = 0$  we must have  $\Theta = 0$ .

The problem thus formulated is one of eigenvalues, in which a relationship between the material parameters  $\sigma, \beta, G$  must be established.

2. Asymptotic Analysis. We will seek an asymptotic solution of the problem formulated where  $\beta, G$  are large. Then, as in [11], the effects of the nonparallel nature of the flow become insignificant. If we take  $G = O(\beta^2)$ , then from analysis of the equations we arrive at the conclusion that  $v = O(\beta^2 u)$ ,  $\Theta = O(u)$ . As in [11], in the asymptotic limit  $\beta \rightarrow \infty$  we find that perturbations are localized inside a thin layer located at a distance  $y_+$  from the wall, while the layer thickness is of the order of  $\beta^{-1/2}$ . The meaning of this inner layer is that therein there is established a balance of centrifugal, convective, and viscous terms of the linearized Navier-Stokes equations. In analogy to [11], we conclude that within the layer it is necessary to seek a solution in the form of expansions

$$\begin{aligned} \widehat{v}(\eta) &= v_0(\eta) + \epsilon^{1/2} v_1(\eta) + \epsilon v_2(\eta) + \dots, & \widehat{u}(\eta) &= \epsilon^2 u_0(\eta) + \epsilon^{5/2} u_1(\eta) + \\ &+ \epsilon^3 u_2(\eta) + \dots, & \widehat{w}(\eta) &= \epsilon^{1/2} w_0(\eta) + \epsilon w_1(\eta) + \epsilon^{3/2} w_2(\eta) + \dots, & \widehat{p}(\eta) &= \end{aligned}$$

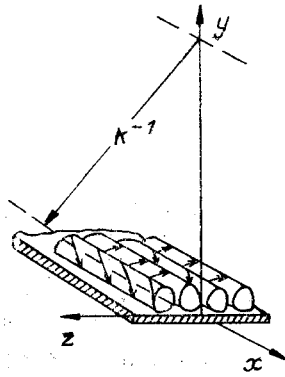


Fig. 1

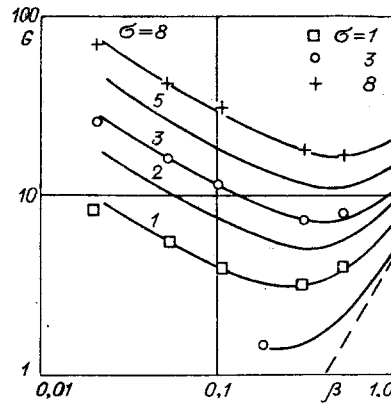


Fig. 2

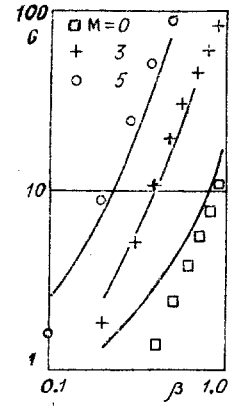


Fig. 3

$$= \varepsilon^{-1/2} p_0(\eta) + p_1(\eta) + \varepsilon^{1/2} p_2(\eta) + \dots, \quad \widehat{\Theta}(\eta) = \varepsilon^2 \Theta_0(\eta) + \varepsilon^{5/2} \Theta_1(\eta) + \\ + \varepsilon^3 \Theta_2(\eta) + \dots, \quad G^2 = \varepsilon^{-4} (g_0 + \varepsilon^{1/2} g_1 + \varepsilon g_2 + \dots), \quad \eta = (y - y_+)/\varepsilon^{1/2}, \\ \varepsilon = \beta^{-1}.$$

We take the rate of intensification  $\sigma$  as given and write  $\sigma = \varepsilon^{-2} \sigma_0$ . Thus,  $\sigma_0$  is a measure of the deviation from the neutral stability curve. In particular,  $\sigma_0$  may be chosen equal to zero. Substituting the expansions written above in the original system of equations for the perturbations and transforming to the variable  $\eta$ , we obtain in the main approximation the system of equations

$$\left( \frac{\sigma_0 U}{T} + \mu \right) u_0 + \frac{v_0}{T} \frac{dU}{dy} = 0, \\ \frac{2Ug_0}{T} u_0 + \left( \frac{\sigma_0 U}{T} + \mu \right) v_0 - \frac{U^2}{T^2} g_0 \Theta_0 = 0, \quad \frac{v_0}{T} \frac{dT}{dy} + \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \Theta_0 = 0, \\ w_0 = -dv_0/d\eta, \quad p_0 = -\mu dv_0/d\eta + \sigma_0 U w_0/T, \quad (2.1)$$

where  $U$ ,  $T$ ,  $\mu$  are the velocity, temperature, and viscosity coefficient in the original flow and  $Pr$  is the Prandtl number. It is assumed that the viscosity is a function of temperature. In Eq. (2.1) and below all functions referring to the fundamental flow as well as derivatives thereof with respect to  $y$  are calculated at a point, the coordinate of which,  $y_+$ , is still unknown. The first three equations of Eq. (2.1) can be separated, and from the condition of their solubility we obtain an expression relating  $y_+$ ,  $g_0$ ,  $\sigma_0$ :

$$\left[ \left( \frac{\sigma_0 U}{T} + \mu \right)^2 - \frac{2g_0 U}{T} \frac{dU}{dy} \right] \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) + \frac{U^2}{T^3} g_0 \frac{dT}{dy} \left( \frac{\sigma_0 U}{T} + \mu \right) = 0. \quad (2.2)$$

All the unknown functions can be expressed in terms of  $v_0$ :

$$u_0 = -\frac{v_0}{T} \frac{dU}{dy} \left/ \left( \frac{\sigma_0 U}{T} + \mu \right) \right., \quad w_0 = -\frac{dv_0}{d\eta}, \\ \Theta_0 = -\frac{v_0}{T} \frac{dT}{dy} \left/ \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \right., \quad p_0 = -\frac{dv_0}{d\eta} \left( \frac{\sigma_0 U}{T} + \mu \right). \quad (2.3)$$

At this stage the function  $v_0(\eta)$  remains undefined. In the next approximation we arrive at the equations

$$\left( \frac{\sigma_0 U}{T} + \mu \right) u_1 + \frac{v_1}{T} \frac{dU}{dy} = F_1(\eta), \\ \frac{2Ug_0}{T} u_1 + \left( \frac{\sigma_0 U}{T} + \mu \right) v_1 - \frac{U^2}{T^2} g_0 \Theta_1 = F_2(\eta), \quad \frac{v_1}{T} \frac{dT}{dy} + \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \Theta_1 = F_3(\eta), \\ F_1(\eta) = -u_0 \eta \frac{d}{dy} \left( \frac{\sigma_0 U}{T} + \mu \right) - v_0 \eta \frac{d}{dy} \left( \frac{1}{T} \frac{dU}{dy} \right), \\ F_2(\eta) = -\frac{2Ug_1}{T} u_0 + \frac{U^2}{T^2} g_1 \Theta_0 - 2g_0 u_0 \eta \frac{d}{dy} \frac{U}{T} - \eta v_0 \frac{d}{dy} \left( \frac{\sigma_0 U}{T} + \mu \right) + 2g_0 \eta \Theta_0 \frac{d}{dy} \frac{U}{T}, \\ F_3(\eta) = -v_0 \frac{d}{dy} \left( \frac{1}{T} \frac{dT}{dy} \right) - \eta \Theta_0 \frac{d}{dy} \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right). \quad (2.4)$$

The condition for solubility of Eq. (2.4) has the form

$$F_2 \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \left( \frac{\sigma_0 U}{T} + \mu \right) + F_3 \frac{U^2}{T^2} g_0 \left( \frac{\sigma_0 U}{T} + \mu \right) - \frac{2Ug_0}{T} \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) F_1 = 0. \quad (2.5)$$

Equation (2.5) contains terms proportional to  $\eta$  and  $g_1$ . If Eq. (2.2) is satisfied, it can be shown that for the validity of Eq. (2.5) it is necessary to take  $g_1 = 0$ . As a result, with the aid of Eqs. (2.2), (2.5) we find  $y_+$ ,  $g_0$  for a given value of  $\sigma_0$ . Substituting in Eq. (2.5) the expressions for  $F_1$ ,  $F_2$ ,  $F_3$  from Eq. (2.4), then using Eqs. (2.3), (2.2), we arrive at an equation defining  $y_+$ :

$$2 \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \left( \frac{\sigma_0 U}{T} + \mu \right) \frac{d}{dy} \left( \frac{U}{T^2} \frac{dU}{dy} \right) - \left( \frac{\sigma_0 U}{T} + \mu \right)^2 \frac{d}{dy} \left( \frac{U^2}{T^3} \frac{dT}{dy} \right) + \frac{U^2}{T^3} \frac{dT}{dy} \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \frac{d}{dy} \left[ \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \left( \frac{\sigma_0 U}{T} + \mu \right) \right] - \frac{4U}{T^2} \frac{dU}{dy} \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \frac{d}{dy} \left( \frac{\sigma_0 U}{T} + \mu \right) = 0. \quad (2.6)$$

If the Mach number  $M$  is equal to zero and  $T = \text{const}$ ,  $\sigma_0 = 0$ , it follows from Eq. (2.6) that  $y_+$  corresponds to the coordinate of the maximum of the function  $U(y)dU/dy$  [11]. In the third approximation we obtain the following system:

$$\begin{aligned} \frac{v_2}{T} \frac{dU}{dy} + u_2 \left( \frac{\sigma_0 U}{T} + \mu \right) &= S_1(\eta), \\ \frac{2Ug_0}{T} u_2 + \left( \frac{\sigma_0 U}{T} + \mu \right) v_2 - \frac{g_0 U^2}{T^2} \Theta_2 &= S_2(\eta), \\ \frac{v_2}{T} \frac{dT}{dy} + \left( \frac{\sigma_0 U}{T} + \frac{\mu}{Pr} \right) \Theta_2 &= S_3(\eta). \end{aligned} \quad (2.7)$$

The expressions for  $S_1$ ,  $S_2$ ,  $S_3$  are given in the appendix. The condition for solubility of Eq. (2.7) coincides with that for Eq. (2.5), it being necessary to replace  $F_j$  in Eq. (2.5) by  $S_j$  ( $j = 1, 2, 3$ ). If using Eq. (2.4) we express  $u_1$ ,  $\Theta_1$  in terms of  $v_1$  and substitute them in the expressions for  $S_1$ ,  $S_2$ ,  $S_3$ , then from the corresponding condition for solubility of Eq. (2.7) we obtain a differential equation for  $v_0(\eta)$ :

$$Ad^2 v_0 / d\eta^2 - \eta^2 B v_0 - g_2 v_0 C = 0. \quad (2.8)$$

The coefficients  $A$ ,  $B$ ,  $C$  appearing in Eq. (2.8) are written out explicitly in the appendix. Following [11], we transform to the variable  $\zeta = \sqrt{2}\eta(B/A)^{1/4}$  and obtain from Eq. (2.8)

$$\frac{d^2 v_0}{d\zeta^2} - \frac{1}{4} \zeta^2 v_0 - \frac{1}{2} g_2 \frac{C}{A} \left( \frac{A}{B} \right)^{1/2} = 0. \quad (2.9)$$

A solution of Eq. (2.9) which decays as  $\zeta \rightarrow \pm\infty$  was indicated in [11]:  $v_0 = \exp(-\zeta^2/4) H_m(\zeta)$  (where  $H_m$  is a Hermitian polynomial). The unknown constant  $g_2$  is then defined by the condition

$$g_2 = \frac{A}{C} \sqrt{\frac{B}{A}} (1 + 2m) \quad (m = 0, 1, 2, \dots). \quad (2.10)$$

The various values of the parameter  $m$  in Eq. (2.10) correspond to various modes of the problem of boundary layer stability on a curved surface. In the present study we will consider only the first mode corresponding to  $m = 0$ . Results of numerical calculations of  $y_+$ ,  $g_0$ ,  $g_2$  for  $\sigma_0 = 0$  and various  $M$  are presented in Table 1. The calculations were performed for a thermally insulated surface at  $Pr = 0.72$  with adiabatic index  $\gamma = 1.4$ . The viscosity was assumed to depend on temperature by the Southerland expression. The braking temperature was 310 K.

**3. Approximate Relationships.** As was shown in [12], asymptotic analysis of the problem as  $\beta \rightarrow \infty$  gives good results for formal use of the expressions obtained for  $\beta \lesssim 1$ . The asymptotic results are convenient in that instead of the complex procedure of solution of systems of differential equations one needs solve only algebraic equations, which can easily be programmed for numerical analysis. It proves that with formal use of the asymptotic relationships for  $\beta = O(1)$ ,  $\sigma \gtrsim 1$  the coordinate  $y_+$  is located at a short distance from the wall. Therefore to obtain approximate relationships one can use an expansion of all functions of the fundamental flow in Taylor series in the vicinity of  $y = 0$ . We will limit ourselves to the first terms of the expansions. Then from Eq. (2.6) for  $Pr = 1$  with the viscosity linearly dependent on temperature

TABLE 1

M	$y_+$	$g_0$	$g_2$
0	2,16	2,96	2,86
1	2,85	4,08	3,62
3	5,11	14,50	10,72

$$\sigma_0 = T_w^2 / U_w' y_+, \quad (3.1)$$

where the prime indicates differentiation with respect to  $y$ , and the subscript  $w$  indicates that the given function is calculated at  $y = 0$ . The proposed approximate relationships will more precisely approximate the asymptotic solutions found when the surface flowed over is thermally isolated and boundary layer suction is absent, since in this case the second term of the expansion in powers of  $y_+$  vanishes. Thus, the error of Eq. (3.1) is of the order  $O(y_+)$ . The approximate relationships are convenient only for  $y_+ \rightarrow 0$ , which imposes a corresponding limitation on  $\sigma_0$  in Eq. (3.1). A precise estimate of the admissible values of  $\sigma_0$  must be determined from consideration of the residual term in Eq. (3.1). As is evident from comparison of numerical calculation results with the approximate relationships, the latter give satisfactory results even for  $\sigma > 1$ .

After all necessary calculations we obtain the following expressions:

$$g_0 = 2T_w^4 / ((U_w')^2 y_+) = 2T_w^2 \sigma_0 / U_w', \quad g_2 = \sqrt{2} T_w^4 / ((U_w')^2 y_+^2) = \sqrt{2} \sigma_0^2, \quad (3.2)$$

$$G^2 = g_0 \beta^4 + g_2 \beta^3 = 2T_w^2 \sigma \beta^2 / U_w' + \sqrt{2} \sigma^2 / \beta.$$

Considering that on a thermally isolated surface without boundary layer suction  $T_w = (1 + 0.5(\gamma - 1)M^2)$ ,  $U_w' = 0.332$ , from Eq. (3.2) we find

$$G = \sqrt{6.02(1 + 0.5(\gamma - 1)M^2)^2 \sigma \beta^2 + 1.41 \sigma^2 / \beta^2}. \quad (3.3)$$

Figure 2 shows a comparison of direct numerical solution of the problem for  $M = 3$  from [10] (solid lines) and of a calculation with approximate equation (3.3). We recall that the approximate expressions (3.1)-(3.3) are inapplicable as  $\sigma \rightarrow 0$ . Figure 2 also shows the asymptote for the neutral stability curve (dashed line). From Eq. (3.3) we can find an equation for the line of maximum intensification rates in the plane  $(\beta, G)$ :

$$G = 12.4 \beta^{5/2} (1 + 0.5(\gamma - 1)M^2)^2. \quad (3.4)$$

Figure 3 shows a comparison of calculation results from [10] (lines) with those obtained by the approximate equation (3.4). It is evident from Figs. 2, 3 that the approximate relationships reflect the direct numerical calculation results properly. Their simplicity makes them useful for approximate calculations as used in engineering practice.

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#### APPENDIX

$$q_1 = \frac{\sigma_0 U}{T} + \mu, \quad q_2 = \frac{\sigma_0 U}{T} + \frac{\mu}{Pr},$$

$$S_1 = -u_1 \eta \frac{dq_1}{dy} - \frac{u_0}{2} \eta^2 \frac{d^2 q_1}{dy^2} - v_1 \eta \frac{d}{dy} \left( \frac{1}{T} \frac{dU}{dy} \right) - \frac{v_0}{2} \eta^2 \frac{d^2}{dy^2} \left( \frac{1}{T} \frac{dU}{dy} \right) + \mu \frac{d^2 u_0}{d\eta^2},$$

$$S_2 = -\frac{2Ug_2 u_0}{T} + \frac{g_2 U^2}{T^2} \Theta_0 - 2g_0 u_1 \eta \frac{d}{dy} \left( \frac{U}{T} \right) - g_0 u_0 \eta^2 \frac{d^2}{dy^2} \left( \frac{U}{T} \right) - v_1 \eta \frac{dq_1}{dy} -$$

$$-\frac{v_0}{2} \eta^2 \frac{d^2 q_1}{dy^2} + g_0 \Theta_1 \eta \frac{d}{dy} \left( \frac{U^2}{T^2} \right) + \frac{g_0}{2} \Theta_0 \eta^2 \frac{d^2}{dy^2} \left( \frac{U^2}{T^2} \right) + \mu \frac{d^2 v_0}{d\eta^2} - \frac{dp_0}{d\eta},$$

$$S_3 = -v_1 \eta \frac{d}{dy} \left( \frac{1}{T} \frac{dT}{dy} \right) - \frac{v_0}{2} \eta^2 \frac{d^2}{dy^2} \left( \frac{1}{T} \frac{dT}{dy} \right) - \Theta_1 \eta \frac{dq_2}{dy} - \frac{\Theta_0}{2} \eta^2 \frac{d^2 q_2}{dy^2} + \frac{\mu}{Pr} \frac{d^2 \Theta_0}{d\eta^2},$$

$$A = \frac{2U}{T^2} \frac{dU}{dy} \mu \frac{q_2}{q_1} - \frac{\mu}{Pr} \frac{U^2}{T^3} \frac{dT}{dy} g_0 \frac{q_1}{q_2} + q_1 q_2 (q_1 + \mu),$$

$$B = -2g_0 \left[ \frac{q_2 U}{q_1 T} \frac{dq_1}{dy} - q_2 \frac{d}{dy} \left( \frac{U}{T} \right) \right] \left[ \frac{1}{q_1 T} \frac{dU}{dy} \frac{dq_1}{dy} - \frac{d}{dy} \left( \frac{1}{T} \frac{dU}{dy} \right) \right] -$$

$$\begin{aligned}
& -g_0 \left[ q_1 \frac{d}{dy} \left( \frac{U^2}{T^2} \right) - \frac{U^2 q_1}{T^2 q_2} \frac{dq_2}{dy} \right] \left[ \frac{1}{q_2 T} \frac{dT}{dy} \frac{dq_2}{dy} - \frac{d}{dy} \left( \frac{1}{T} \frac{dT}{dy} \right) \right] - \\
& - \frac{g_0}{T} q_2 \frac{dU}{dy} \frac{d^2}{dy^2} \left( \frac{U}{T} \right) + \frac{q_1 q_2}{2} \frac{d^2 q_1}{dy^2} + \frac{g_0 q_1}{2T} \frac{dT}{dy} \frac{d^2}{dy^2} \left( \frac{U^2}{T^2} \right) + \frac{g_0 U^2 q_1}{2T^2} \frac{d^2}{dy^2} \left( \frac{1}{T} \frac{dT}{dy} \right) - \\
& - \frac{g_0 U^2 q_1}{2T^3 q_2} \frac{dT}{dy} \frac{d^2 q_2}{dy^2} + \frac{g_0 U q_2}{T q_1} \frac{dU}{dy} \frac{d^2 q_1}{dy^2} - g_0 \frac{U}{T} q_2 \frac{d^2}{dy^2} \left( \frac{1}{T} \frac{dU}{dy} \right), \\
& C = \frac{U^2 q_1}{T^3} \frac{dT}{dy} - \frac{2U q_2}{T^2} \frac{dU}{dy}.
\end{aligned}$$

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